Profit Maximization, Industry Structure, and Competition: A critique of neoclassical theory

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Abstract

Neoclassical economics has two theories of competition between profit-maximizing firms—Marshallian and Cournot-Nash—that start from different premises about the degree of strategic interaction between firms, yet reach the same result, that market price falls as the number of firms in an industry increases. The Marshallian argument is strictly false. We integrate the different premises, and establish that the optimal level of strategic interaction between competing firms is zero. Simulations support our analysis and reveal intriguing emergent behaviors.

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1 A popular fallacy

The underlying assumption of the Marshallian model is that the i^{th} firm in a competitive industry does not react strategically to the hypothetical actions of other firms. In an n firm industry where the output of the i^{th} firm is q_i, this assumption, known as "atomism" or "price-taking"[3, pp. 314, 383], means that $\frac{\partial q_i}{\partial q_j} = 0 \; \forall i \neq j$. This model then claims that the market demand function $P(Q)$ (where $Q = \sum_{i=1}^{n} q_i$) has the dual properties that $P'(Q) < 0$ and $P'(q_i) = 0$ for large n. Elementary calculus shows this is false [5, p. 8, footnote 31.], because $\frac{dQ}{dq_i} = 1$ if $\frac{\partial q_i}{\partial q_j} = 0 \; \forall i \neq j$, so that:

$$\frac{dP}{dq_i} = \frac{dP}{dQ}\frac{dQ}{dq_i} = \frac{dP}{dQ} < 0 \tag{1}$$

The false belief that $\frac{dP}{dq_i} = 0$ is essential to the Marshallian derivation of the model of "perfect competition", which occurs when $P(Q)$—the marginal benefit to society—equals the marginal cost of production $MC(Q)$ [3, p. 322]. The derivation starts from the proposition that a firm will maximize its profit $\pi(q_i)$ by producing where the gap between its total revenue $(P(Q)\,q_i)$ and total cost $(TC(q_i))$ is greatest. Neoclassical economics asserts that this output level can be identified by equating "Marginal Cost" $\left(MC(q_i) = \frac{d}{dq_i}TC(q_i)\right)$ and "Marginal Revenue" $\left(MR(q_i) = \frac{d}{dq_i}(P(Q)\,q_i)\right)$:

$$\pi_{neo} : \frac{d}{dq_i}\pi(q_i) = MR(q_i) - MC(q_i) = 0 \tag{2}$$

Given this alleged profit-maximization rule and the "assumption" that $P'(q_i) = 0$, it followed that for perfect competition, price equalled marginal cost. Since the assumption is logically incompatible with $P'(Q) < 0$, the Marshallian derivation of perfect competition fails.

The Cournot-Nash model is not dependent on this fallacy, arguing instead that strategic interactions lead to a Nash equilibrium in which market price converges to marginal cost as the number of firms increases ([6]; [3, pp. 411-413.])—though this process is, at best, highly conditional (see [3, pp. 417-423]). Standard neoclassical analysis assumes that firms *will* strategically interact, and calculates the i^{th} firm's best response on this basis under various conditions. We instead treat $\frac{\partial q_i}{\partial q_j}$, the response of the i^{th} firm to a hypothetical change in output by the j^{th}, as a decision variable, and consider what is its optimal value of $\frac{\partial q_i}{\partial q_j}$ for a profit-maximizing firm. As a preliminary, we show that the proposition that (2) maximizes profits for the i^{th} firm is false.

2 The true profit maximization formula

In a multi-firm industry, the profit maximum is given by the zero, not of its *partial* derivative, but its *total* derivative—since the actions of other firms affect the profitability of any given firm, even though (or rather, especially because) the i^{th} firm cannot control what the other firms in the industry do. Attempting to maximize profit, while ignoring what other firms do, is rather like trying to row a boat to a specific location, while ignoring the wind and tides. The profit maximum for the i^{th} firm is therefore given by:

$$\frac{d}{dQ}\pi(q_i) = \frac{d}{dQ}(P(Q)\,q_i - TC(q_i)) = 0 \tag{3}$$

Expanding this expression in terms of reaction coefficients $\frac{\partial q_i}{\partial q_j}$, and setting $\frac{\partial q_i}{\partial q_j} = 0 \forall i \neq j$ with the Marshallian assumption of "atomism", equation (3) reduces to:[1]

$$P + nq_i \frac{dP}{dQ} - MC\left(q_i\right) = 0 \tag{4}$$

This contradicts the neoclassical belief, epitomized by (2), that, in the context of atomistic behavior, profit is maximized by equating marginal revenue and marginal cost, since $n = 1$ only in the case of a monopoly—which is the one time that the accepted Marshallian formula is correct. At all other times, the profit maximum for an individual firm will occur where "marginal revenue" *exceeds* "marginal cost". As a consequence, the Marshallian model of atomistic behavior leads to industry output being independent of the number of firms in it, in contradiction of standard neoclassical pedagogy and belief.

In Cournot-Nash game theoretic analysis, firms decide their own actions on the basis of the expected reactions of other firms, in such a way that each firm's "best response" is to set $MR\left(q_i\right) = MC\left(q_i\right)$. In our terms, this is equivalent to setting $\frac{\partial q_i}{\partial q_j} = \dfrac{1}{nE}$—where E is the market elasticity of demand ($E = -\dfrac{P}{Q}\dfrac{dQ}{dP}$). We instead treat $\frac{\partial q_i}{\partial q_j}$ as a decision variable whose optimum value can be identified by the firm. Consider an industry with n identical firms (a common heuristic device in economic theory) so that $\frac{\partial q_i}{\partial q_j} = \theta \ \forall i \neq j$ (and $\frac{\partial q_i}{\partial q_i} = 1$), where θ can take on any value. Then (3) reduces to:

$$\left(n-1\right) P\theta + P + nq_i \frac{dP}{dQ} = MC\left(q_i\right) \tag{5}$$

This defines the maximum profit achievable by the individual firm in the context of strategic behavior—if each firm reacts to output changes by other firms with a reaction coefficient of θ. The optimum value for θ for the i^{th} firm occurs where $\frac{d}{d\theta}\pi\left(q_i\right) = 0$. This condition reduces to:

$$\frac{1}{n}\frac{d}{d\theta}Q\left(P + nq_i \frac{dP}{dQ} - MC\left(q_i\right)\right) = 0 \tag{6}$$

[1] These derivations are rather intricate and require much more space than is available here. For full details, please consult *www.debunking-economics.com/totf*.

Since it can be shown that $\frac{d}{d\theta}Q \neq 0$, (6) equals zero iff $P + nq_i\frac{dP}{dQ} - MC(q_i) = 0$. As established above, this requires that $\theta = 0$. Firms thus achieve higher profits if they *do not* react strategically to each other. In the classic words of the movie *War Games*, firms may conclude that Cournot-Nash strategic interaction is "A curious game. The only winning strategy is not to play." We consider this question using a multi-agent model of instrumentally rational profit maximizers facing comparable marginal cost functions.

3 Operationally rational profit-maximizers

Our hypothetical market has a linear demand curve ($P(Q) = a - bQ$) and a given number n of profit-maximizing agents. Firm i chooses an initial output level q_i, and a fixed amount by which to vary output at each step δq_i. If profit—the gap between revenue $P(Q)\,q_i$ and total cost $tc(q_i, n)$, defined below—falls after an iteration, firm i reverses the sign of δq_i for the next iteration. [2] Total cost functions for the firms are identical. The polynomial form used gives rising marginal cost, as assumed by neoclassical theory—though the empirical norm is constant or falling marginal cost [1, Chapter 4], [2, pp. 119-121]—and ensures that aggregate marginal cost is comparable across different industry structures: [3]

$$tc(q_i, n) = k + Cq_i + \frac{1}{2}Dnq_i^2 + \frac{1}{3}En^2q_i^3 \tag{7}$$

In the following simulations, $a = 800$ and $b = 10^{-7}$, $k = 10^6$, $C = 10$, $D = 10^{-8}$and $E = 10^{-17}$ and n ranges between 5 and 100 (higher values made no significant difference to our results); the fixed δq_i for each firm is drawn from a normal distribution $N(m, \sigma)$ where $m = 0$ and σ is a given fraction of the Cournot prediction. Monte Carlo simulations reveal a rich range of interactions, and in general show that instrumentally rational profit-maximizers will learn "not to play" the Cournot-Nash game. For low σ, output converges to the "Keen" equilibrium given by (4) for all values of n (Figure 1).

However, as σ rises from 1% to 20% of the Cournot prediction, the outcome shifts from the Keen to the Cournot level (Figure 2). Aggregate and individual agent behavior also becomes much more unstable, as Figure 3 indicates. We surmise that the "emergent collusion" we identified in [4] breaks down as δq rises; perhaps the increasing size of unpredictable output changes by other

[2] The programs for this paper can be downloaded from *www.debunking-economics.com/totf.*

[3] Conventional neoclassical exposition assumes this is a general rule, but it in fact applies only under very limited conditions [2, pp. 121-122]

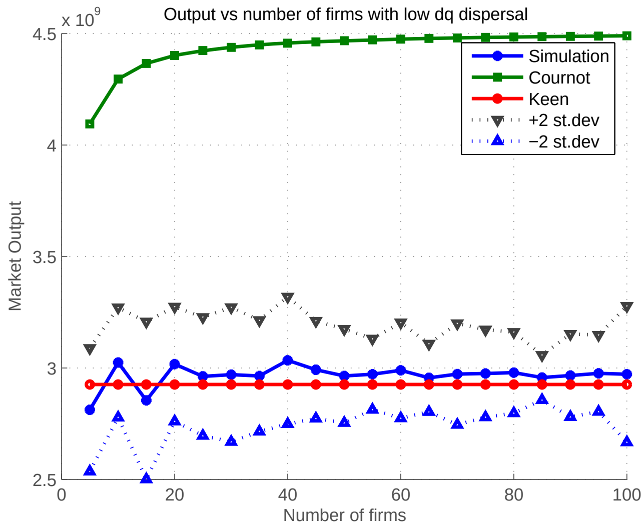

Fig. 1. Outcome of Monte Carlo simulations with low σ (2% Cournot)

firms makes the overall market environment more chaotic, forcing each firm to rely more on feedback from its own output.changes.

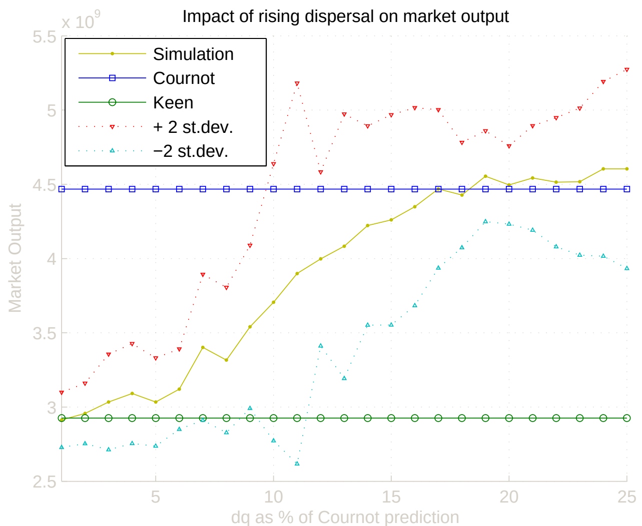

Fig. 2. Convergence to Cournot-Nash prediction as σ rises with constant $n = 50$

4 Conclusion

Contrary to the beliefs of the vast majority of economists, equating marginal revenue and marginal cost is not profit-maximizing behavior, the number of firms in an industry has no discernible impact on the quantity produced, price exceeds marginal cost in "competitive" industries, the "deadweight loss of welfare" exists regardless of how many firms there are in the industry, and

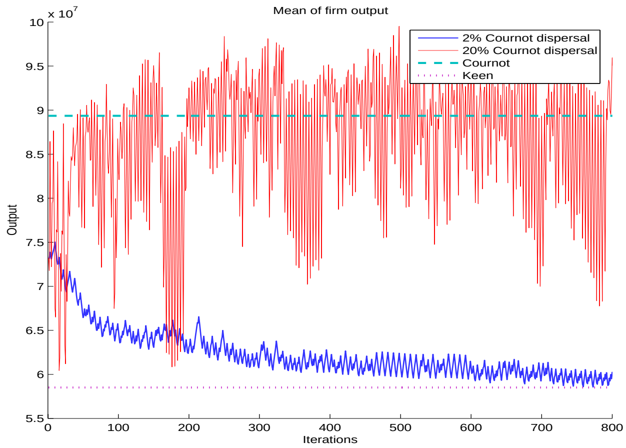

Fig. 3. Comparative stability of average firm for different values of σ

instrumentally rational profit-maximizers do not play Cournot-Nash games. Moving from Hollywood to The Bard, it appears that the dominant neoclassical theory of the firm is “Much Ado About Nothing”.

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