A simple approach to modelling endogenous money

Steve Keen[1]

1 Introduction

One issue in Heterodox economics on which there is widespread agreement is that the money supply is endogenous—in contrast to the Neoclassical convention of treating the money supply as exogenous (and somehow under the control of the Central Bank). Basil Moore's pioneering work inaugurated this consensus, with a persuasive verbal and diagrammatic account of the method by which money is created—via the lines of credit that major corporations have negotiated with their banks (Moore 1983: 544, 546; see Rochon 2001 for a longer historical perspective). More recently, the Italian-French Circuit School provided a sound "first principles" perspective on the process of money creation in a pure credit economy (Graziani 1989, 2003).

However, while a verbal consensus exists, there is not yet any accepted *mathematical* model of how money is endogenously created by the financial-corporate system. Wynne Godley and Marc Lavoie have done the most sophisticated work to date (Godley 1999; Godley & Lavoie 2007), using the Social Accounting Matrix (*SAM*) framework that Godley pioneered, and there have been many other papers inspired by this approach. Several Circuit theorists have also attempted to model the dynamics of endogenous money, sometimes employing the *SAM* framework (Bellofiore et al. 2000; Fontana 2003). However, many conundrums exist in this literature—notably, Circuit theorists are perplexed by the issue of whether profits exist in the aggregate (Bellofiore et al 2000: 410; Rochon 2005: 125; Messori et al.2005).

In this paper, I argue for a related paradigm to Godley and Lavoie's, but with five substantial methodological and substantive differences. I then present a deliberately stylized model, which nonetheless captures the essential aspects of endogenous money creation, and provides unexpected answers to several endogenous money conundrums.

2 System design

My framework, which I call a Monetary Accounting Matrix (*MAM*), is closely related to Godley's *SAM* approach, but differs in five major ways:

1. Time is modelled continuously using differential equations, rather than discretely using difference equations;
2. The model system states are bank accounts, with assets and liabilities modelled separately;
3. Wage, profit and rentier incomes are not aggregated;
4. The equivalent to the "Social Accounting Matrix", the "Monetary Accounting Matrix" (*MAM*), does not have the restrictions that are applied to Godley's *SAM*. In particular, though I apply a similar principle to "Each row and column of the flow matrix sums to zero on the principle that every flow comes from somewhere and goes somewhere" (Godley 1999: 394; see also Godley & Lavoie 2007: 9 *et alia*):

[1] Associate Professor of Economics & Finance, University of Western Sydney, Australia; www.debunkingeconomics.com; www.debtdeflation.com/blogs

4.1. The columns of the *MAM* do not sum to zero, but instead return the differential equations of the model; and

4.2. When endogenous money growth is introduced, *the rows of the MAM sum to more than zero*.

5. There is no "n[th] equation rule", as in the Godley *SAM* framework..

2.1 Continuous time

A continuous time formulation is inherently superior to the discrete time approach used by most *SAM* modellers, on at least four grounds.

- Firstly, while every individual economic transaction, like every birth, is a discrete event, these transactions, also like births, are dispersed through time. Aggregate economic processes are thus better captured by continuous time equations—as indeed is the aggregate of birth and death processes in biology, radioactive decay, etc.[2]

- Secondly, the time dependencies in discrete time models often force unrealistic compromises on the modeller—as Godley noted when he observed that "I have introduced lags ... whenever simultaneous interdependence threatened to generate meaningless oscillations" (Godley 1999: 409). No such problem exists in a continuous time model—in part because of the third, and major, advantage of this approach over discrete time modelling.

- In a continuous time model, *all* entries are flows. In a discrete time formulation, while most entries are flows, some are stocks (for example, Table 1 in Godley 1999 lists eleven flow variables and six stocks), and stocks have to be explicitly linked with flows via equations of the form $Stock_{t+1} = Stock_t + Flows_t$. Stocks in a continuous time model are the value of its system states, which are given by the integral of the flows. There is thus no danger of mis-specifying a stock as a flow—a perennial problem in economic modelling.[3] Nor is there a danger of not properly linking a flow to a stock variable: once a flow is introduced into the model, it is automatically linked to the appropriate stock via differential equations of the form $\frac{d}{dt} Stock(t) = \sum Flows(t)$.[4]

- Finally, time dependencies are much more easily handled in continuous time form. For example, consumption should be subject to a much shorter time delay that investment; however doing so in a discrete time frmaework would mean having difference equations of the form $C_{t+1} = F(Y_t)$ for consumption and $I_{t+52} = F(Y_t)$ for investment (indicating a week's lag for consumption and a year's lag for investment). In practice, to make their models tractable, researchers frequently use the same time delays for variables that beat to a very different drum, leading to serious distortions of the underlying dynamics. No such problem exists with continuous time modelling, where very different time lags can easily be mixed—and they can even be variable.

[2] Andresen has proven this as a theorem using systems engineering analysis (Andresen 1998).

[3] Godley and Lavoie 2007 opens with the wonderful remark by Kalecki that economics "is the science of confusing stocks with flows" (Godley & Lavoie 2007: 1)

[4] Time lags and time delays are also easily introduced into this formulation, though none are employed in this paper's model.

2.2 Transaction accounts rather than economic entities

The transactions approach is partly a product of the Circuitist origins of this model, which are discussed in more detail below. However, a crucial advantage of having transaction accounts as the fundamental system states—rather than aggregate economic agent like "Households"—is that the actual financial transactions of the system are explicitly shown, and separated from physical transfers. *This includes the endogenous creation of money*: in a credit money system, money is created in bank accounts, and the transactions paradigm allows this to be modelled directly. In contrast, even in Godley's sophisticated *SAM* framework, the modelling of endogenous money creation is implicit rather than explicit.

The transactions paradigm is also the basis of the next three points of difference between my work and Godley's.

2.3 No household sector

In most *SAM*-based work, profits from firms, net interest income from financial transactions, and wages are aggregated into the income of the household sector (see again Table 1 of Godley 1999). I avoid this step, largely on the basis that the behavior of different social classes is different, and this is lost by aggregating all classes into the amorphous unit "households". The transactions accounts basis of the model facilitates this disaggregated approach.[5]

2.4 Absence of column-row restrictions

Columns do not sum to zero

The construction of the model is also simplified: since the columns sum the flows into and out of any given transaction account (and the dynamics of the record of debt), *the model's differential equations are derived simply by adding up the columns*. The differential equation approach also allows a natural modelling of both in and out of equilibrium states.

A net positive MAM

As is discussed in more detail when growth is introduced, the chief reason that the *MAM* with growth has a net positive sum is that *not all entries in a matrix of bank accounts are in fact transactions* (that a debt account is, strictly speaking, not an account at all, but a record of financial obligations between borrower and lender, is also significant here).

When Moore's "line of credit" arrangement is introduced, an entry occurs in the credit account of firms for which there is a matching entry in firms' record of debt, *but there is no matching transaction transfer from any other account*. This is the source of endogenous money growth, which is explicitly modelled in this framework.

2.5 No n^{th} equation rule

As Godley and Lavoie remark, the fact that the n^{th} equation is determined by the other *n-1* equations in their models is related to the same rule that applies in Walrasian economics, where the n^{th} market's equilibrium is required if the other *n-1* markets are in equilibrium. This feature arises from the mixed stock-flow nature of the SAM framework. No such closure rule is required

[5] Class-based differential ownership of firms and banks can be modelled by introducing equity accounts, with shares being purchased by flows from bank accounts.

in differential equation models, where the model is fully specified by its flows and a set of initial conditions—of which there is one per system state (or stock).

3 Basic Circuitist model

The Circuitist model of money, as developed by Graziani (1989, 2003), starts from a definition of money as a token whose transfer is accepted as final payment in all exchanges.[6] This in turn necessitates that all payments are made "by means of promises of a third agent, the typical third agent being nowadays a bank" (Graziani 1989: 3), where in turn the bank is so constrained that it cannot profit from seignorage. All exchanges therefore involve a single commodity or service, and three parties: a seller, a buyer, and a bank that records payment as a transfer from the bank account of the buyer to that of the seller.

The canonical Circuit model of a pure credit production economy has three classes—bankers, capitalists, and workers. Capitalists own firms that produce output which is sold to other capitalists, workers and bankers; bankers lend money at interest to capitalists to finance production; and workers work in factories to produce output for capitalists.[7]

At the minimum, three transaction accounts are needed—one each for the deposits of capitalists, workers and bankers—while one record of debt is also needed, to record any lending from bankers to capitalists.[8]

I indicate the deposit accounts as F_D, B_D and W_D respectively, and the Firm Loan account as F_L. Prior to the making of a loan, all four accounts have zero balances. This "ab initio" situation is shown in Table 1, along with the sum of the Monetary Accounting Matrix.

Bank Accounts					
	Assets	**Liabilities**			MAM
Accounts	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Initial Condition	0	0	0	0	0

Table 1: Initial conditions prior to loan

In step one of the model, banks make loans to the firms. Since this is credit money, a debt obligation is created between the firms and banks along with the creation of money. Using L to signify the magnitude of the loan, this results in the situation shown in Table 2—and though we have not yet begun to consider the flows initiated by the loan, the positive sum of the *MAM* at this stage is instructive.[9]

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[6]Rochon (2001) makes a strong case that this approach to money was also present in Kahn and Robinson's work.

[7]There is no Central Bank in the base model: money is therefore *exclusively* credit money. There is no Central-Bank-created fiat money—though this can easily be introduced into the MAM framework later—and a Central Bank is needed later when the banking sector is "exploded" to allow inter-bank transactions. In the model in this paper, the banking sector is modelled as a single unit.

[8]In this paper, I abstract from the physical flows to focus solely on the financial dynamics—in particular, the process by which money is endogenously created. Implicitly of course, there are physical flows of commodities moving in the opposite direction to financial flows.

Bank Accounts					
	Assets	Liabilities			MAM
Accounts	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Loan	L	L	0	0	"L"

Table 2: Loan issued

Godley and Lavoie may feel that this row should indeed be balanced by treating the entry in F_L as a negative, and summing across both assets and liabilities. I believe that this convention encourages the belief that a loan is in effect negative money, so that a debt is repaid by transferring money into it. However, a loan account is **not** a bank account as such: it does not contain money—let alone negative money—nor can money be paid into it.[10]

Instead, a loan account is a record of debt, and when a money deposit is made into a debtor's deposit account, an equivalent entry is made on the record of debt. But money is not and cannot be paid into it, whereas money is paid into, and transferred between, Deposit Accounts. The *MAM* is the aggregate of all money transfers, which at this stage of the model is the sum of the Bank's liabilities only.

Alternatively, it could be argued that the +L in the F_D account should balanced by a -L in the B_D account, so that overall the MAM summed to zero. However, the essence of endogenous money from the Circuitist point of view is that the "promises to pay" of the bank are accepted as full payment in any commercial transaction. The bank's promise alone is therefore all that needs to be entered into the F_D account to constitute money: this is why the deposit constitutes a liability of the bank. A loan is not a commodity transaction, but a grant of purchasing power in return for a debt obligation. The balance in this financial transaction is provided by the matching entry in the assets column, which specifies that the firm now has a debt to the bank.[11]

We now consider the obligations the loan imposes. Once the loan has been issued, it generates an obligation to pay interest to the lender, while the deposit obligates the bank to pay interest to the depositor. I use r_L for the rate of interest on loans and r_D for the rate on deposits, (where $r_L>r_D$). These obligations are shown in Table 3.

Bank Accounts					
	Assets	**Liabilities**			MAM
Accounts	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Obligations	$+r_L \cdot F_L$	$+r_D \cdot F_D$	0	0	N/A

Table 3: Loan and deposit obligations

[9]It is shown in inverted commas because this is an initial condition, rather than a flow, and the model is constituted by its flows.

[10]I explore the consequences of using the Godley-Lavoie convention later in the paper.

[11]This point may become clearer when continuous creation of endogenous money is introduced in Table 9.

Finally we start the model itself by considering the flows initiated by the loan. For the loan obligations to be met, flows must occur out of accounts in the system—since there is no other source of money. The firms must therefore pay the loan interest obligation out of their deposit account F_D, while the bank must pay its deposit interest obligation out of its deposit account B_D.

The flows occur between these two deposit accounts, while the payment of loan interest is recorded on the asset side of the ledger, resulting in the debt stabilising at the level of the initial loan L (I consider repayment of the loan principal later). Since the interest payments flow between the firm and banker deposit accounts, the overall sum of deposit accounts also stabilises at L. But since $r_L > r_D$, the balance shifts from the firms deposit account to the bankers over time. This dynamic is shown in Table 4.

Bank Accounts					
	Assets	Liabilities			MAM
Accounts	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Flows: Interest	$+r_L \cdot F_L$ $-r_L \cdot F_L$	$+r_D \cdot F_D$ $-r_L \cdot F_L$	$-r_D \cdot F_D$ $+r_L \cdot F_L$	0	0

Table 4: Payment of interest

Equation 1 states this incomplete system as a set of coupled ODEs—which are derived simply by adding up the entries in each column. Thus the rate of change of the Firms' Deposit account ($\frac{d}{dt}F_D(t)$) is $+r_D \cdot F_D - r_L \cdot F_L$:

1)
$$\begin{aligned} \tfrac{d}{dt}F_L &= 0 \\ \tfrac{d}{dt}F_D &= r_D F_D - r_L F_L \\ \tfrac{d}{dt}B_D &= r_L F_L - r_D F_D \\ \tfrac{d}{dt}W_D &= 0 \end{aligned}$$

The same balance that is apparent in Table 4 applies here: by inspection it is obvious that the level of debt will remain constant (at the initial value L), as will the sum of deposit accounts, but the money in the firms' account will be transferred to the banks'. At some point, firms' deposit accounts will turn negative—which is of course an unsustainable situation.

Figure 1 shows a simulation of this system. Given the set of example parameter values (L=100, r_L=5%, r_D=3%) while the outstanding loan and the sum of deposit accounts remain at 100 throughout, all the money has been transferred from the firms' deposit account to the bankers' after 30.5 years.

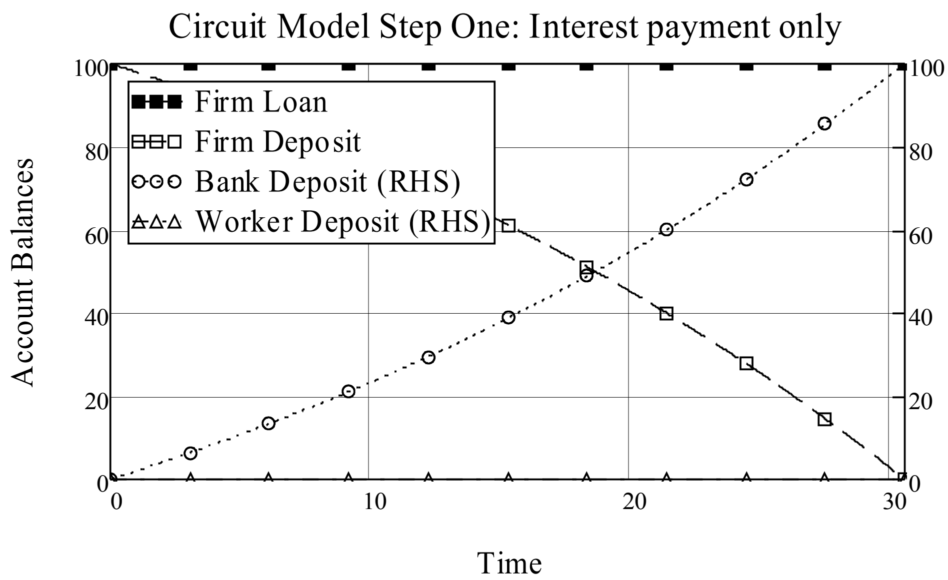


Figure 1: Simulation of interest payment only

The next flow considered is the payment of wages by firms, which causes an outflow from the firms deposit account of the amount $w \cdot F_D$ and a corresponding inflow into the workers deposit account. This flow is transferred per unit of time (per year in this model) from firms to workers as wages.

Bank Accounts					
	Assets	Liabilities			MAM
Flows	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Interest	$+r_L \cdot F_L$ $-r_L \cdot F_L$	$+r_D \cdot F_D$ $-r_L \cdot F_L$	$-r_D \cdot F_D$ $+r_L \cdot F_L$	0	0
Wages	0	$-w \cdot F_D$	0	$+w \cdot F_D$	0

Table 5: Spending to finance production

With workers now having positive bank balances, they too are receipients of interest income. Though in the real world workers normally get lower deposit rates than firms, for simplicity I will use the same rate of interest r_D here. A flow of $r_D.W_D$ is therefore deducted from the bankers' account, and deposited into the workers' account.

Bank Accounts					
	Assets	**Liabilities**			MAM
Flows	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Interest	$+r_L \cdot F_L$ $-r_L \cdot F_L$	$+r_D \cdot F_D$ $-r_L \cdot F_L$	$-r_D \cdot F_D$ $+r_L \cdot F_L$	0	0
Wages	0	$-w \cdot F_D$	0	$+w \cdot F_D$	0
Interest	0	0	$-r_D \cdot W_D$	$+r_D \cdot W_D$	0

Table 6: Workers Interest Income

To complete the model, we have to include the flow of transactions from workers and bankers to capitalists that purchase the goods flowing (implicitly in this model) in the opposite direction. Here I use ω for the rate at which spending flows from workers' deposit accounts to firms', and β for the corresponding rate of spending by banks. The amounts $\omega.W_D$ and $\beta.B_D$ are therefore deducted from workers and banks accounts respectively and credited to the firms' account.

The basic model is finally complete, and the components of the basic coupled ODE model can now be read down the columns of Table 7.

Bank Accounts					
	Assets	**Liabilities**			MAM
Flows	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Interest	$+r_L \cdot F_L$ $-r_L \cdot F_L$	$+r_D \cdot F_D$ $-r_L \cdot F_L$	$-r_D \cdot F_D$ $+r_L \cdot F_L$	0	0
Wages	0	$-w \cdot F_D$	0	$+w \cdot F_D$	0
Interest	0	0	$-r_D \cdot W_D$	$+r_D \cdot W_D$	0
Purchases	0	$+\omega \cdot W_D + \beta \cdot B_D$	$-\beta \cdot B_D$	$-\omega \cdot W_D$	0

Table 7: Transactions complete the basic model

In coupled ODE form, the model is as shown in Equation 2.

2)

$$\begin{aligned}
\tfrac{d}{dt} F_L &= 0 \\
\tfrac{d}{dt} F_D &= \left(r_D F_D - r_L F_L\right) - w \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) \\
\tfrac{d}{dt} B_D &= \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
\tfrac{d}{dt} W_D &= w \cdot F_D + r_D \cdot W_D - \omega \cdot W_D
\end{aligned}$$

The model can now be simulated (see Figure 2; the additional parameter values used here—the values for which are explained later—are w=3, ω=26 and β=0.5).[12] Since it is a linear model, its equilibrium can also be derived symbolically (see equation 3)

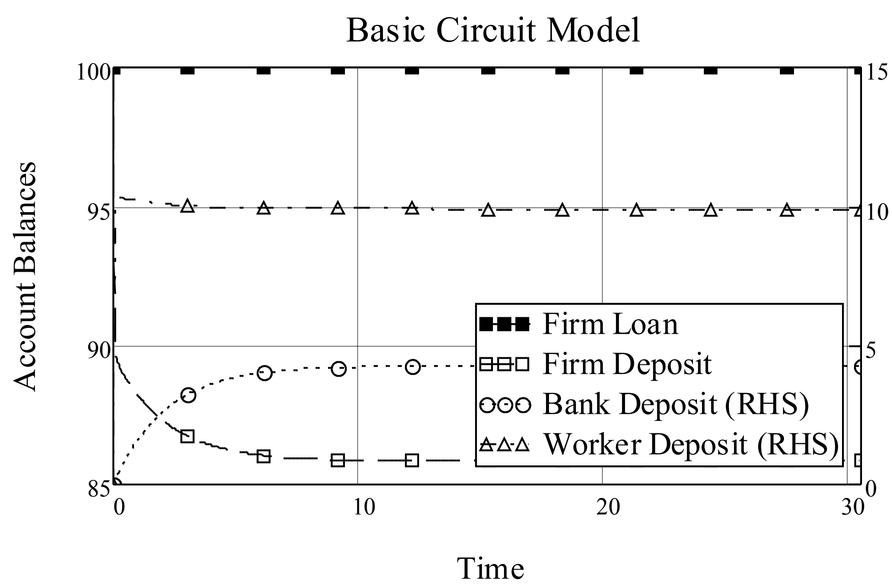


Figure 2: Basic Circuit model

As is now obvious, the basic Circuitist model with a single injection of endogenous money is consistent with sustained economic activity over time. Contra Graziani 2003 and most Circuitist attempts to model endogenous money, an increasing supply is *not* needed to sustain constant economic activity (this confirms the result in Andresen 2006).

$$3)\quad \begin{bmatrix} F_{L_E} \\ F_{D_E} \\ B_{D_E} \\ W_{D_E} \end{bmatrix} = \begin{bmatrix} L \\ \dfrac{L\cdot(\omega - r_D)\cdot(\beta - r_L)}{(w+\omega - r_D)\cdot(\beta - r_D)} \\ \dfrac{L\cdot(r_L - r_D)}{\beta - r_D} \\ \dfrac{L\cdot w\cdot(\beta - r_L)}{(w+\omega - r_D)\cdot(\beta - r_D)} \end{bmatrix} = \begin{bmatrix} 100 \\ 85.83 \\ 4.255 \\ 9.915 \end{bmatrix}$$

However the amounts shown here are transaction account balances: we do not yet know whether these are compatible with sustained incomes over time.

3.1 Income dynamics

Fortunately, two income flows are easily associated with particular transactions in equation (3): wages and interest income. Annual wages are equal to $w\cdot F_D$ and bank interest income is $r_L F_L$ (which equal 257.489 and 5 per annum respectively in this simulation). Wages and interest income are thus positive and sustained in this model; what about profits?

[12]The values of these variables are explained in section , where time lags are introduced.

To reveal profits, we need to consider what the term *w* represents. As well as being a determinant of wages, it also represents that part of the net surplus from production that accrues to workers. The net surplus—in monetary terms—itself depends on how rapidly money invested in production returns to firms. In Marx's terms, it represents the time lag between extending M and receiving M+ (assuming, as I do in this skeletal model, that the process occurs smoothly).

This could be a period of, say, 4 months between financing production and receiving the complete proceeds of sale of output—again, something that would be a variable in a more complex model. There are thus two components to *w*: the share of the net surplus from production going to workers (in Sraffa's sense of the surplus, in which wages and profits are entirely paid out of the net surplus from the input-output production process), and the rate of turnover from M to M+, given by technical conditions of production and the time taken for the sale of physical commodities. I use *s* for the share of surplus accruing to the owners of firms (so that the share going to workers is thus *1-s*), and *P* for the lag between M and M+.[1] We therefore have the relation given by equation :

4) $$w = (1 - s) \cdot P$$

With *w* set to 3 in the simulation above, a hypothetical value of *s* of 0.4 (which corresponds to a "rate of surplus value", in Marx's terms, of 67%) yields a value for *P* of 5 (which means that the lag between spending M and making M+ is 1/5th of a year or 2.4 months). The monetary value of net output per annum is thus $P.F_D$ (which equals 429.15 in equilibrium, given the parameter values in the model) which is split between workers and the owners of firms in the ratio *(1-s):s*. In this debt-finance only model, the owners of firms then have to pay interest on their outstanding debt to banks. Using Π, W and I to signify profits, wages and interest income respectively, the income flows of the model in equilibrium are:

5) $$\begin{bmatrix} \Pi_E \\ I_E \\ W_E \end{bmatrix} = \begin{bmatrix} s \cdot P \cdot \dfrac{L \cdot (\omega - r_D) \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ r_D \cdot L \\ (1-s) \cdot P \cdot \dfrac{L \cdot (\omega - r_D) \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \end{bmatrix} = \begin{bmatrix} 166.66 \\ 5 \\ 257.49 \end{bmatrix}$$

Firms thus do make net profits, which, though related to the size of the initial loan, can be substantially larger than this amount (and profits are substantially larger than the servicing cost of debt).

The size of the initial loan L can also be related to the equilibrium value of wages generated by the loan:

6) $$L = W_E \cdot \frac{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)}{(1-s) \cdot P \cdot (\omega - r_D) \cdot (\beta - r_L)} = 100$$

Two more issues remain to be considered: the impact of debt repayment, and growth.

3.2 Debt repayment and bank reserves, and the non-destruction of money

According to Graziani—and almost all theorists in endogenous money, including Godley & Lavoie—the repayment of debt destroys the money that was created with it (Graziani 2003: 29-30). I consider this by adding an additional term R_L to represent the repayment of loans. If we relate this to the level of outstanding debt[2], then the amount $R_L.F_L$ is deducted from the firms' only source of money, F_D. Yet to where does it go?

Here Graziani's third condition for the existence of money comes into play: "the use of money must be so regulated as to give no privilege of seigniorage to any agent" (Graziani 2003: 60). This repayment therefore cannot be made to the existing bankers' deposit account B_D, since banks use this account to finance spending on commodities. It must therefore go to a separate, capital account: the banks' reserve account, which I call B_R.

Reserves, once created by the repayment of loans, will be relent. This amount will be deducted from the banks' reserve account and deposited in the firms' deposit account—and a matching entry will be made in the firms loan record of account. The complete relations are shown in Table 8.

Bank Assets & Liabilities					
Time	Assets	Liabilities			MAM
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)	Income
Repayment of debt	$-R_L.F_L$	$-R_L.F_L$	0	0	$-R_L.F_L$
Relending of reserves	$+L_R.B_R$	$+L_R.B_R$	0	0	$+L_R.B_R$

Bank Reserves		
Time	Reserve Account	Capital
Repayment of debt	$R_L.F_L$	$+R_L.F_L$
Relending of reserves	$-L_R.B_R$	$-L_R.B_R$
	MAM	0

Table 8: Repayment and relending

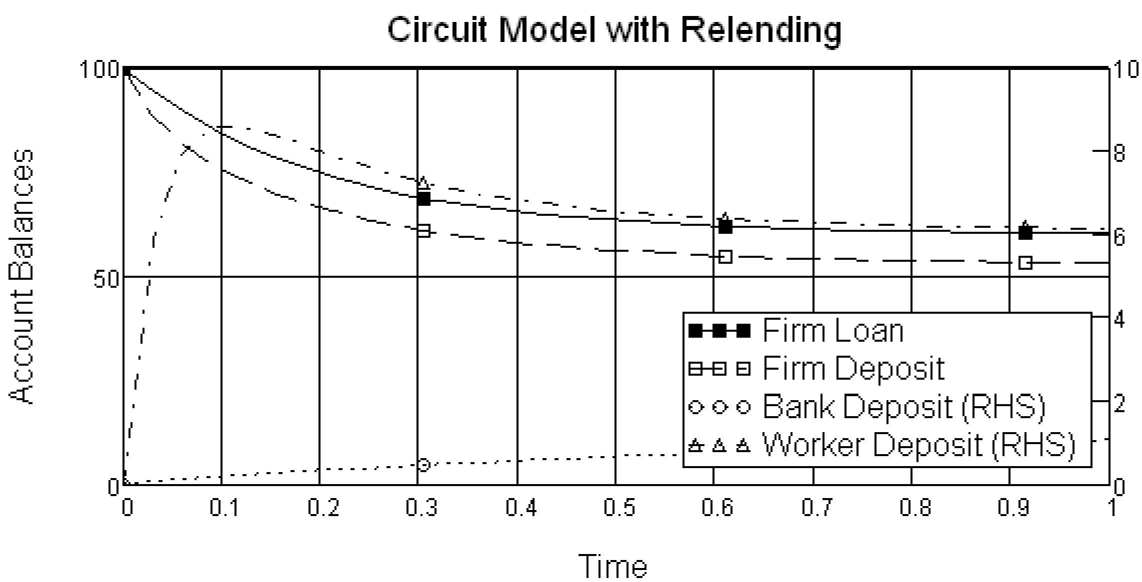
The repayment of loans therefore does not "destroy" money, but transfers it out of income accounts—where it can be used for expenditure—to a reserve account. Once there, it is an unemcumbered asset of the banks which can then be re-lent—though not spent directly on commodities or services. This adds an important additional insight to the concept of endogenous money: not only do "loans create deposits", but "the repayment of loans creates reserves".[13]

This results in the model shown in equation (7):

[13] Rochon 2001 shows that Robinson & Kahn reached a similar position in their consideration of the endogeneity of money

$$
\begin{aligned}
&\tfrac{d}{dt} F_L = +L_R \cdot B_R - R_L \cdot F_L \\
&\tfrac{d}{dt} F_D = \left(r_D F_D - r_L F_L\right) - \left(1 - s\right) \cdot P \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) + \left(L_R \cdot B_R - R_L \cdot F_L\right) \\
7) \quad &\tfrac{d}{dt} B_D = \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
&\tfrac{d}{dt} W_D = \left(1 - s\right) \cdot P \cdot F_D + r_D \cdot W_D - \omega \cdot W_D \\
&\tfrac{d}{dt} B_R = +R_L \cdot F_L - L_R \cdot B_R
\end{aligned}
$$

The simulation results for this model are shown in Figure 3 (with a shorter time span to show the initial dynamics). The new parameters R_L and L_R were given the values of 2 and 3 respectively.



Deposit Accounts $F_D(Y) = 51.5$ $B_D(Y) = 2.55$ $W_D(Y) = 5.95$ $F_D(Y) + B_D(Y) + W_D(Y) = 60$

Bank Assets $F_L(Y) = 60$ $B_R(Y) = 40$ $F_L(Y) + B_R(Y) = 100$

Income Flows $s \cdot P \cdot F_D(Y) = 103$ $(1 - s) \cdot P \cdot F_D(Y) = 154.49$ $r_L \cdot F_L(Y) = 3$

Figure 3: Model with repayment and relending

The equilibrium values are shown in Equation :

$$8)\qquad \begin{bmatrix} F_{L_E} \\ F_{D_E} \\ B_{D_E} \\ W_{D_E} \\ B_{R_E} \end{bmatrix} = \frac{1}{L_R + R_L} \cdot \begin{bmatrix} L_R \cdot L \\ L_R \cdot \dfrac{L \cdot (\omega - r_D) \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ L_R \cdot \dfrac{L \cdot (r_L - r_D)}{\beta - r_D} \\ L_R \cdot \dfrac{L \cdot (1-s) \cdot P \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ R_L \cdot L \end{bmatrix} = \begin{bmatrix} 60 \\ 51.5 \\ 2.55 \\ 5.95 \\ 40 \end{bmatrix}$$

It is obvious that money is not destroyed, but turned into reserves that are then available for relending. However there is a reduction in money in circulation at any one time, equivalent to the proportion of debt that has been repaid. Given the parameters used in this simulation, the amount of circulating money is reduced from 100 to 60 units.

It is thus not money that is "destroyed" by the repayment of debt, but deposits in income accounts. This in turn reduces the amount available for the financing of production, reducing all incomes—including that of banks. The equilibrium levels of income are now:

$$9)\qquad \begin{bmatrix} \Pi_E \\ I_E \\ W_E \end{bmatrix} = \begin{bmatrix} 103 \\ 3 \\ 159.49 \end{bmatrix}$$

3.3 Growth

The final topic is how to model endogenous growth in the money supply. As Moore argues, firms negotiate "lines of credit" with banks that enable them to expand the available money, subject to the same sum being added to their outstanding debt. New money is thus created by an addition of an identical sum to the firms' deposit and loan accounts

Using F_I (for "Firms' Investment") to signify the rate, and relating this to the level of firms' deposit accounts,[3] this introduces a new term $F_I.F_D$ into the columns for F_L and F_D in the final table. I have included the creation and simultaneous transfer of this new money in the banks' reserve account simply to indicate that the endogenous creation of money by firms depends upon the legal right they have negotiated with banks to expand their borrowings.[4]

Bank Assets & Liabilities					
Time	Assets	Liabilities			MAM
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)	Income
Investment by firms	$+F_I.F_D$	$+F_I.F_D$	0	0	$+F_I.F_D$

Bank Reserves		
Time	Reserves	Capital
Investment by firms	$+F_I.F_D - F_I.F_D$	0

Table 9: Endogenous creation of new money

There is no offsetting transfer between income and capital accounts in this case, so that the term $F_I.F_D$ causes a net increase in the money stock: it is the source of endogenous money. As a result, rather than having a zero sum, the complete *MAM* has a positive sum equal to the amount of new money $F_I.F_D$ being created each year. The overall model, as shown in Equation 10, is therefore "dissipative"—in the language of modern dynamic analysis—rather than "conservative", which has important implications for the feasible behavior of the complete model that will be built on this skeleton.

$$
\begin{aligned}
&\tfrac{d}{dt}F_L = +L_R \cdot B_R - R_L \cdot F_L + F_I \cdot F_D \\
&\tfrac{d}{dt}F_D = \left(r_D F_D - r_L F_L\right) - \left(1-s\right)\cdot P \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) + \left(L_R \cdot B_R - R_L \cdot F_L\right) + F_I \cdot F_D \\
10)\ &\tfrac{d}{dt}B_D = \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
&\tfrac{d}{dt}W_D = \left(1-s\right)\cdot P \cdot F_D + r_D \cdot W_D - \omega \cdot W_D \\
&\tfrac{d}{dt}B_R = +R_L \cdot F_L - L_R \cdot B_R
\end{aligned}
$$

Though the amount of money and debt in this final model grow exponentially over time, the same relations hold between debt and income deposits, while the overall money stock includes both the sum of deposit accounts and the amount in banks' reserves. At the end of the simulation period (30 years), the endogenous money stock has grown from 100 to 379.13, 228.78 of which is in circulation between firm, bank and worker income accounts, and 150.35 of which is in the banks' reserve account.

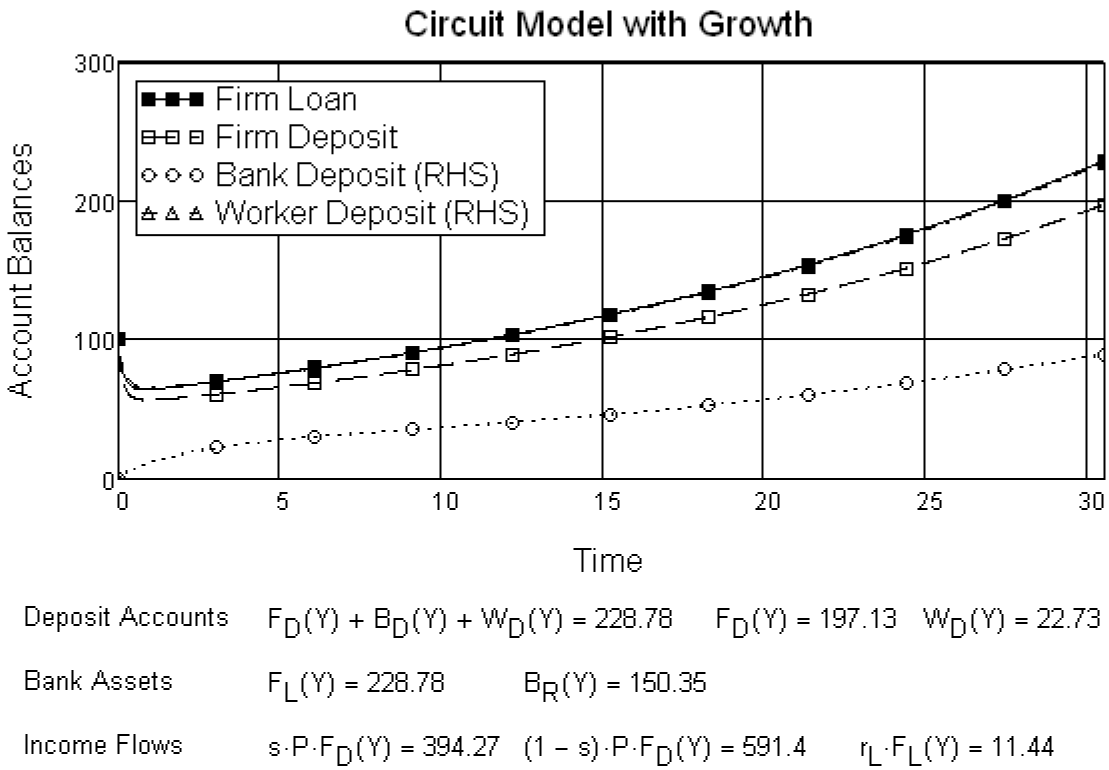


Figure 4: Model with growth

3.4 Speculations on Government and Fiat Money

I have not yet extended the model to incorporate a government and central bank, but in principle it is easy to do so. Two additional "bones" need to be added to the skeleton: a government fiscal account, and a central bank. The government's taxes and expenditures and the central bank's "lender of last resort" functions would then generate fiat money, in addition to the credit money created by the banks and firms.

The fiscal component of the skeleton is relatively straightforward: taxes levied on all income-earning classes would constitute the inflows into the government fiscal account, while outflows would include purchases from firms to support the state apparatus.

The fiat money aspect is rather more subtle. The essential difference between credit and fiat money is that the former involves the creation of an initially identical debt obligation for the recipient, whereas the latter appears as an asset without a corresponding offsetting liability *for the recipient*. The liability instead appears as a debt of the government to the central bank.

The creation of fiat money also has two components: that initiated by the funding of any government deficit,[5] and that initiated by actions by the central bank to ensure systemic stability.

3.5 *MAM* vs *SAM* Speculations

One substantial difference between my *MAM* framework and other attempts to model endogenous money creation, is my argument that repayment of debt does not "destroy" money;

instead, the debt repayment effectively creates bank reserves (Rochon 2001 argues that this proposition was also made by Robinson & Kahn).

Part of my argument here stems from insisting that debt is *not* "anti-money". As well as meaning that money is not automatically annihilated by debt repayment, it also necessitates an account into which money given to banks in repayment of debt can be deposited—hence the reserve account.

One way to consider the standard argument that debt repayment does destroy money is to set the MAM up so that debt is treated as anti-money, and then analyse the dynamics of this system. This is done in Table 10: F_L is now a negative number, and repayment is now shown as being an addition of an amount $+R_L.F_L$ to the F_D account (which is negative since F_L is negative) and the subtraction of an equivalent sum from F_L (a positive amount, since F_L is negative), which therefore reduces debt by an equivalent amount.

Similarly, new investment by firms—accessing their lines of credit and thus endogenously creating money—adds an equivalent negative sum to debt in the final row of the Table. There is no B_R account, since money intended for debt repayment is now paid into the F_L account and "annihilated".

This model accurately captures the conventional "debt repayment destroys money" perspective. Reading down the columns of Table 10 yields the system of equations shown in (11) below (when simulated, the initial loan L is now shown as a positive entry in F_D and a negative entry in F_L)

Firstly, the model without new money is no longer inherently "conservative" in the complex systems sense: the sum of the MAM will be negative if debt repayment occurs (i.e., if parameter $R_L>0$); it will be positive if new money creation exceeds money destruction by debt repayment (i.e., if $+F_I.F_D >+R_L.F_L$).

Bank Accounts					
	Assets	Liabilities			MAM
	Firm Loan F_L	Firm Deposit F_D	Bank Deposit B_D	Worker Deposit W_D	
Initial condition	0	0	0	0	0
Loan	-L	+L	0	0	0
Obligations	$+r_L.F_L$	$+r_D.F_D$	0	0	0
Flows					
Interest	$+r_L \cdot F_L$ $-r_L \cdot F_L$	$+r_D \cdot F_D$ $+r_L \cdot F_L$	$-r_D \cdot F_D$ $-r_L \cdot F_L$	0	0
Wages	0	$-w \cdot F_D$	0	$+w \cdot F_D$	0
Interest	0	0	$-r_D \cdot W_D$	$+r_D \cdot W_D$	0
Purchases	0	$+\omega \cdot W_D + \beta \cdot B_D$	$-\beta \cdot B_D$	$-\omega \cdot W_D$	0
Repayment	$-R_L.F_L$	$+R_L.F_L$	0	0	$+R_L.F_L$
Investment by firms	$-F_I.F_D$	$+F_I.F_D$	0	0	$+F_I.F_D$

Table 10: Debt as Anti-Money

$$
\begin{aligned}
&\frac{d}{dt}F_L = -R_L \cdot F_L - F_I \cdot F_D \\
&\frac{d}{dt}F_D = \left(r_D F_D + r_L F_L\right) - w \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) + R_L \cdot F_L + F_I \cdot F_D \\
&\frac{d}{dt}B_D = \left(-r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
&\frac{d}{dt}W_D = w \cdot F_D + r_D \cdot W_D - \omega \cdot W_D
\end{aligned} \quad (11)
$$

Secondly, though real-world finance is clearly destabilizing, to borrow two famous expressionsfrom Kaldor, this model displays either "dangerous instabilities", or "more stability than the real world appears, in fact, to possess" (Kaldor 1940: 80). With no new money creation, economic activity fizzles out within a year (using the same parameter values that generated sustained economic activity in the "debt repayment does not destroy money" model above).

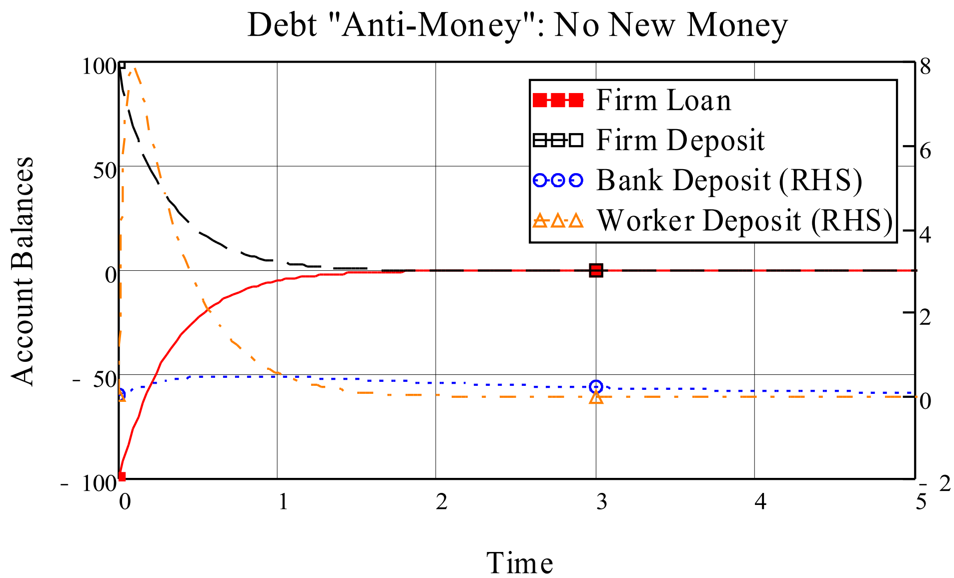


Figure 5: Debt as Anti-Money: No New Money

With the new money creation parameter F_I 15% larger than the loan repayment parameter R_L, the economy is roughly stable indefinitely; at 20% larger, the system explodes—the money supply grows by a factor of 38 over 30 years.

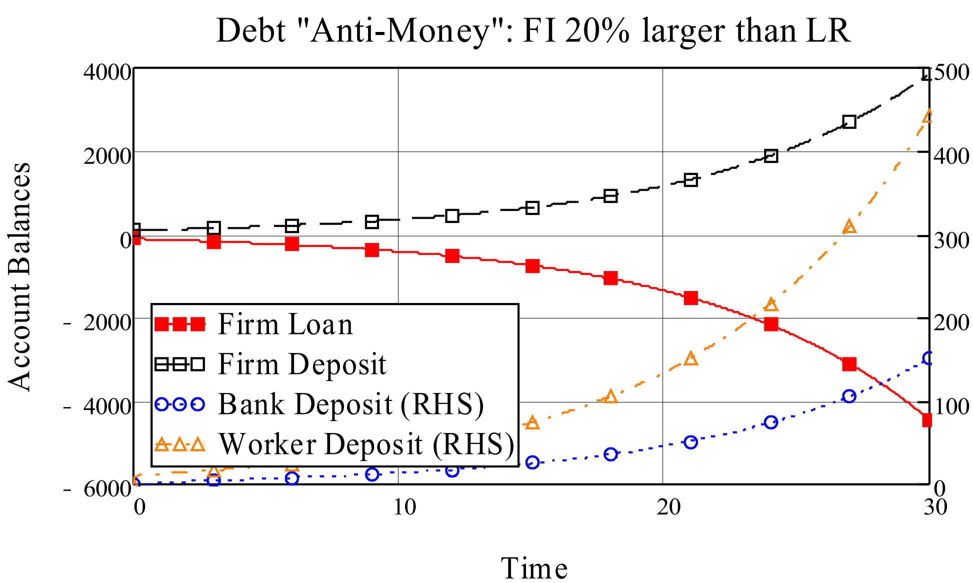


Figure 6: Debt as Anti-Money: Explosive growth of money supply

These alternate simulations indicate that, while the proposition that debt repayment does not destroy money is unconventional, it may in fact be correct, and the heterodox "conventional wisdom" may be false.

3.6 Meaning of parameters

It may seem strange that parameters like w, ω and β have values greater than one. This is because they are actually inverse time-lags: the value of 26 for ω, for example, indicates that workers spend their wages every two weeks. In proper "system dynamics" form, the parameter would have been $\tau_W = 1/\omega$, and would have been equal to 1/26—since two weeks is 1/26th of a year, and a year is the time unit of this model.

3.7 Conclusion

The *MAM* framework provides a simple means to model what has been an agreed but contentious issue within heterodox economics: the mechanism by which money is endogenously created. The framework's results contradict several widely accepted views in heterodox economics—that debt repayment destroys money, that aggregate profits cannot be made, and that new money must continuously be created to sustain constant economic activity—but in ways that strengthen the overall heterodox approach. I invite other heterodox researchers to give it a workout.

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3.9 Index

[1] Again, in a more complete model, each of these stages of the process would have their own equation with its own dynamics; here, for reasons of simplicity and exposition, they are all collapsed into the values of s and P.

[2]It could equally be related to the level of F_D.

[3] It could as easily be related to the level of outstanding loans, and would doubtless have a more complex causal link in a full dynamic model.

[4] In a full model, this could be given a rationing ceiling; however I believe that a better way to indicate banks' "structuralist" control over lending is to replace R_L with a variable dependent upon financial conditions.

[5] Financing of a government deficit by the issuance of bonds can also be accommodated within this framework., with of course a different impact upon the money supply.